বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : DSE 1B/2B/3B - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

[MECHANICS]

1. Answer any ***five*** questions : $2\times5=10$

(a) If a particle describes a circle of radius 'a' about the pole, then what will be its radial and transverse velocities?

(b) If the tangential and normal accelerations of a particle moving in a plane curve are equal, then find an expression for its velocity.

(c) What do you understand by impulse of a force and an impulsive force?

(d) Define angle of friction and cone of friction.

(e) If a particle moves in a straight line and its velocity v at a distance x from the origin is given by $v^2 = 2 - x - x^2$. Show that the motion is simple harmonic and find its amplitude.

(f) A particle describes a path $r = ae^{\theta}$ with constant angular velocity. Show that its transverse acceleration varies as the distance from the pole.

(g) What is the work done by the gravity on a particle of mass m lbs during its t^{th} second of its fall.

(h) A particle describes a curve $s = c\tan\psi$ with uniform speed v. Find the acceleration indicating the direction.

2. Answer any ***four*** questions : 5×4=20

(a) Find the C.G. of the arc of the Cardiod $r = a(1+\cos\theta)$ lying above the initial line.

(b) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Show that the C.G. of the remainder is at a distance $\frac{a}{8\pi - 4}$ from the centre of circle of radius $\frac{a}{2}$.

(c) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane, and the plane face is horizontal by a weight P attached at a point in the rim. Prove that the co-efficient of friction is $\frac{P}{\sqrt{W(2P+W)}}$.

(d) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3(u+v)x}{2(u^2+uv+v^2)}$.

(e) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of the recoil of the gun is $\sqrt{\frac{2m^2gh}{M(M+m)}}$.

(f) A particle of mass m moves on a straight line under an attraction mn^2x towards a point 'O'. On the line, where x is the distance from 'O'. If $x = a$ and $x = u$ where $t = 0$, show that the amplitude of the motion is $\sqrt{a^2 + \frac{u^2}{n^2}}$.

3. Answer any ***three*** questions : 10×3=30

(a) A particle of mass m is projected into the air with velocity u in a direction making an angle α with the horizontal. Find the path of the particle, time of flight and maximum horizontal range. 6+2+2

(b) (i) An elastic string whose natural length is equal to that of a rod is attached to the rod at both ends and suspended by its middle point. The rod is of negligible weight and carries a weight W at its middle point. The system starts from rest when the string and the rod are horizontal. Show that the rod will sink until the strings are inclined to the horizon at an angle θ given by the equation $\cot^3\left(\frac{\theta}{2}\right) - \cot\left(\frac{\theta}{2}\right) = 2W$, where the modulus of elasticity of the string is hW.

(ii) Prove that the work-done in raising a body up a smooth inclined plane is the same as the work-done in lifting the body through the vertical height of the plane. 6+4

(c) (i) A thin straight smooth tube is made to revolve upwards with a constant angular velocity ω in a vertical plane about one extremity O. When it is in a horizontal position, a particle is at rest in it at a distance 'a' from the fixed end O. Find the distance of the particle from O after any time t.

Show further that if ω is very small, the particle will reach O in time $\left(\frac{6a}{g\omega}\right)^{\frac{1}{3}}$.

(ii) A curve is described by a particle having a constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 7+3

(d) (i) Prove that any system of co-planar forces acting on a rigid body can be reduced to a single resultant force acting at any arbitrary point in the plane, together with a single resultant couple of moment which is equal to the algebraic sum of moments of the given forces about the arbitrary point.

(ii) The couple components of a system of co-planar forces when reduced w.r. to two different bases O and O' are G and G' respectively. Show that the couple component when the system is reduced w.r. to the middle point of OO' is $\frac{1}{2}(G+G')$. 7+3=10

(e) (i) State the laws of statistical friction. Find the condition of equilibrium of a particle constrained to rest on a rough curve under any given forces.

(ii) A rough wire which has the shape of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is placed with its x-axis vertical and y-axis horizontal. If μ be the co-efficient of friction, find the depth below the heighest point of the position of limiting equilibrium of a bead which rest on the wire. (2+3)+5

OR

[LINEAR PROGRAMMING PROBLEM]

1. Answer any ***five*** questions : 2×5=10

   (a) Show that a hyper plane is a convex set.

   (b) Define extreme point. How many extreme points contains in a straight line?

   (c) Show that the set of all feasible solutions of a LPP is a convex set.

   (d) Show that the dual of dual is primal.

   (e) What is the difference between basic solution and basic feasible solution?

   (f) What do you mean by cycling in LPP? How it can be resolved?

   (g) Find the extreme points if any of the set $S=\left\{(x_1,x_2): x_1^2+x_2^2\leq 1,\ x_1\geq 0,\ x_2\geq 0\right\}$.

   (h) Show that the vectors {(2, 2, 8), (1, 0, 4), (1, 2, 4)} are linearly dependent.

2. Answer any ***four*** questions : 5×4=20

   (a) Use graphical method to solve the following LPP

   Maximize $Z=5x_1+4x_2$

   Subject to $6x_1+4x_2\leq 24;\ x_1+2x_2\leq 6;\ -x_1+x_2\leq 1;\ x_2\leq 2;\ x_1,\ x_2\geq 0$

   (b) Prove that $x_1=5,\ x_2=0,\ x_3=-1$ is a basic solution of the set of equations

   $x_1+2x_2+x_3=4;\ 2x_1+x_2+5x_3=5.$

   Find other basic solution if any.

   (c) Prove that a basic feasible solution to a linear programming problem corresponds to an extreme of the convex set of feasible solutions.

   (d) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and cost 12 per gram. Food Y contains 8 units and 12 units of A and B per gram respectively and costs 20 per gram. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above problem as an LPP to minimize the cost.

(e) Prove that a feasible solution x^* to a primal problem is optimal if and only if there exists a feasible solution y^* to the dual problem such that $cx^* = b^T y^*$, c, b^T have their usual meaning.

(f) Find the dual of the following LPP :

Minimize $Z = x_1 + x_2 + x_3$

$$\text{Subject to} \quad \begin{array}{ccccccc} x_1 & - & 3x_2 & + & 4x_3 & = & 5 \\ x_1 & - & 2x_2 & & & \leq & 3 \\ & & 2x_2 & - & x_3 & \geq & 4 \end{array}$$

$x_1,\ x_2 \geq 0, x_3$ is unrestricted in sign.

3. Answer any ***three*** questions : 10×3=30

(a) Using Simplex Method solve the following LPP

Minimize $Z = -2x_1 + 4x_2 + x_3$

Subject to $x_1 + 2x_2 - x_3 \leq 5;$

$2x_1 - x_2 + 2x_3 \geq 1;$

$x_1,\ x_2,\ x_3 \geq 0$

(b) Find the optimal solution of the following LPP by solving its dual :

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to $x_1 + x_2 \geq 2;$

$x_2 + x_3 \geq 5;$

$x_1,\ x_2,\ x_3 \geq 0$

(c) Find the basic feasible solutions of the following set of equations :

$2x_1 + 6x_2 + 2x_3 + x_4 = 3;$
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2;$
$x_1, x_2, x_3, x_4 \geq 0$

(d) Solve the following LPP

Minimize $Z = x_1 + x_2$

Subject to $x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \geq 48$

$x_1,\ x_2 \geq 0$

(e) Solve the following LPP using Two phase method

Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$,

$x_1 + 7x_2 \geq 7$,

$x_1, x_2 \geq 0$

OR

[NUMERICAL METHODS]

1. Answer any ***five*** questions : 2×5=10

   (a) Round off the numbers of three significant figures (i) 0.00029417 (ii) 49.3628

   (b) When Gauss-Seidal iteration method converge?

   (c) Prove that $\mu^2 = 1 + \frac{1}{4}\delta^2$

   (d) Write down the error term in Newtons forward Interpolation formula.

   (e) Compare between Newton-Cote's quadrature and Gaussian quadrature.

   (f) Find $\Delta^2 u_n$ if $u_n = 2n+1$.

   (g) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$.

   (h) If *f(x)* is a quadratic polynomial, deduce that

   $$\int_1^3 f(x)\,dx = \frac{1}{12}\left[f(0) + 22f(2) + f(4)\right]$$

2. Answer any ***four*** questions : 5×4=20

   (a) Describe Euler's method for solving the differential equation $\frac{dy}{dx} = f(x, y)$ in a finite interval [*a*, *b*] assuming that *y(a)* has a known value y_0.

   (b) Use Runge-Kutta method of order four to approximate *y*, when *x* = 0.1, given that $\frac{dy}{dx} = x + y,\ y(0) = 1$.

   (c) Prove that the sum of Lagrangian function or coefficients is unity.

   (d) Find the condition of convergence of fixed point iteration method.

(e) Derive Newton's backward difference interpolation formula for equi-spaced arguments.

(f) Given a table of values of $f(x)=0$ for $x=x_0+ih(h>0)$, $r=0, 1, 2,...., n$. Find the formula for numerical computation of $f'(x)$, when x is closed to x_n.

3. Answer any ***three*** questions : 10×3=30

(a) Establish Lagrange's interpolation formula for a set of (n+1) points. Prove that the Lagrangian function are invariant under a linear transformation of the independent variable. Outline its merits and demerits as compared to other interpolation formula?

(b) Describe LU factorization Method. Solve the following system system of equations by LU-factorization method

$$x-5y+z=2$$
$$2x+4y+z=1$$
$$x+y+z=0$$

(c) Explain the Newton-Raphson method and its geometrical significance. Apply this to find the approximate root of the equation $x^3-x-1=0$, between $x=1$ and $x=2$, correct to four decimal places.

(d) Write down the approximate representation of $\frac{22}{7}$ correct to four significant figures and then find : (i) Absolute error (ii) relative error (iii) percentage error. Estimate the missing term from the table

x	2	4	6	8	10
y	5	13	*	53	*

. Establish the relation between forward and backward difference operator.

(e) Define the degree of precision in Numerical integration. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using (I) Trapezoidal rule (II) Simpson's one third rule taking $n=6$ and compare with its exact solution.

OR

[INTEGER PROGRAMMING AND THEORY OF GAMES]

1. Answer any ***five*** questions : 2×5=10

(a) Define and explain mixed integer programming problem.

(b) Write the drawback of branch and bound technique.

(c) What is a saddle point?

(d) Explain zero-sum game.

(e) Define : (i) Strategy (ii) Payoff.

(f) Explain some of the practical applications of integer programming.

(g) Show that whatever may be the value of a, the game with the following payoff matrix is strictly determinable :

		B	
		I	II
	I	3	7
A	II	–3	a

(h) Solve the following payoff matrix :

		Player B		
		1	2	3
	I	2	4	5
Player A	II	10	7	9
	III	4	5	6

2. Answer any ***four*** questions : 5×4=20

(a) Players A and B play a game in which each has three coins a 50 paise, a Rs. 1 and a Rs. 2. Each player selects a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, A wins B's coin; if the sum is even, B wins A's coin. Find the best strategy for each player and the value of the game.

(b) Write both primal and dual linear programming problems corresponding to the following rectangular game :

Y

		y_1	y_2	y_3	y_4
	x_1	5	7	4	10
X	x_2	4	3	7	9
	x_3	1	2	5	6

(c) Graphically show that the following problem has no feasible integer solution :

Maximize $z = 2x_1 + 2x_2$

$10x_1 + 10x_2 \leq 9$

Subject to $10x_1 + 5x_2 \geq 1$

$x_1,\ x_2 \geq 0$ and integers

(d) Prove that if we add a fixed number *P* to each element of the payoff matrix, then the optimal strategies remain unchanged while the value of the game is increases by *P*.

(e) A game has the payoff matrix $A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$. Show that expected payoff $E(x, y) = 1 - 2y\left(x - \frac{1}{2}\right)$ and deduced that in the solution of the game, the second player follows a pure strategy while the first has infinite number of mixed strategies.

(f) Solve the following game by graphical method

Player B

		y_1	y_2
Player A	x_1	15	17
	x_2	20	15

3. Answer any ***three*** questions : 10×3=30

(a) A refrigeration and air-conditioning have been awarded a contract for the air-conditioning of a new computer installation. The company has to choose between two alternatives : (i) hire one or more refrigeration technicians for six hours a day or (ii) hire one or more part-time refrigeration apprentice technicians for four hours a day. The rate of wages of a refrigeration technician is Rs. 20 per hour, while the corresponding rate of apprentice technician is Rs. 8 per hour. The company wants to engage the technicians on work for not more than 25 manhours per day and also limit the charges to technicians to Rs. 400. The company estimates that the productivity of a refrigeration technician is eight units and that of a part-time apprentice technician is three units. Formulate the integer programming problem to enable the company to select the optimum number of technicians and apprentices. Hence, solve the corresponding integer programming problem to determine the optimum number of technicians and apprentices.

(b) Solve the following mixed integer programming problem by using Gomory's method :

Maximize $z = 4x_1 + 6x_2 + 2x_3$

$4x_1 - 4x_2 \le 5$

$-x_1 + 6x_2 \le 5$

Subject to $-x_1 + x_2 + x_3 \le 5$

$x_1, x_2, x_3 \ge 0$; x_1, x_3 are integers.

(c) Formulate the following game as a linear programming problem and hence solve it.

		B 1	B 2	B 3
A	1	4	1	–3
A	2	3	1	6
A	3	–3	4	–2

(d) Reduce the following game by dominance method and find the game value :

		Player B				
		I	II	III	IV	V
	I	3	5	4	9	6
Player A	II	5	6	3	7	8
	III	8	7	9	8	7
	IV	4	2	8	5	3

(e) Use branch and bound technique to solve the following integer programming problem :

Maximize $z = x_1 + x_2$

Subject to $4x_1 - x_2 \leq 10$

$2x_1 + 5x_2 \leq 10$

$4x_1 - 3x_2 \leq 6$

$x_1, x_2 \geq 0$ and all are integers.